

Fermions in the vortex core in chiral superconductors.

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Abstract

Using the semiclassical approach to the energy levels of the fermions bound to the vortex core, we found the difference between the states in nonchiral and chiral superconductors, determined by the Berry phase. The bound states of fermions in the singly quantized vortex in the layered superconductor with the symmetry of superfluid $^3\text{He-A}$ is $E = n\omega_0$ (as distinct from $E = (n + 1/2)\omega_0$ in s -wave superconductor) and thus contains the state with exactly zero energy. This is in accordance with the result obtained in microscopic theory [1]. Using this approach for calculations of the effect of impurities on the spectrum of bound state we reproduced the Larkin-Ovchinnikov result for single impurity in s -wave vortex [2, 3]: the spectrum has the double period $\Delta E = 2\omega_0$ and consists of two equidistant sets of levels. The same approach however shows that the single impurity does not change the spectrum $E = n\omega_0$ in the p -wave vortex .

1 Introduction

The low-energy fermions bound to the vortex core play the main role in the thermodynamics and dynamics of the vortex state in superconductors and Fermi-superfluids. The spectrum of the low-energy bound states in the core of the axisymmetric vortex with winding number $N = \pm 1$ in the isotropic model of s -wave superconductor was obtained by Caroli, de Gennes and Matricon [4]:

$$E_n = -N\omega_0 \left(n + \frac{1}{2} \right) , \quad (1)$$

This spectrum is two-fold degenerate due to spin degrees of freedom. The integral quantum number $n = L_z$ is a modified angular momentum of the bound state fermions, while the level spacing ω_0 is the magnitude of the angular velocity of the fermions orbiting about the vortex axis. The direction of rotation is determined by the sign of the winding number N of the vortex.

The level spacing is small compared to the energy gap of the quasiparticles outside the core, $\omega_0 \ll \Delta$. So, in many physical cases the discreteness of n can be neglected. In such cases the spectrum crosses zero energy as a function of continuous angular momentum L_z . So, one has the fermion zero modes. The fermions in this 1D "Fermi liquid" are chiral: the positive energy fermions have a definite sign of the angular momentum n .

The motion of the vortex perturbs the fermions in the vortex core and leads to the spectral flow of the fermionic levels from the negative energy vacuum to the positive energy world of excitations forming the heat bath, or the normal component. This spectral flow leads to the momentum exchange between the vortex core and the heat bath and thus to the additional force acting on the vortex. when it moves with respect to the heat bath [5, 6, 7, 8]. This force was calculated in the microscopic theory in the earlier papers by Kopnin and coauthors [9].

Later the other types of vortices have been found to exist in superfluid ^3He and in high-temperature superconductors: with different winding numbers and in the nonsymmetric environment and/or with the spontaneously broken symmetry in the core. The microscopic description of the bound-state fermions in such distorted vortices becomes complicated. So one needs in the simple phenomenological theory, which describes the low-energy motion of the quasiparticles in the vortex core in the same manner as the low-energy fermions are described in the Landau theory of the Fermi liquid.

Such theory was constructed in the papers [7, 10, 11]. In this approach the fast radial motion of the fermions in the asymmetric vortex core is integrated out and one obtains only the slow motion corresponding to the fermions zero modes. The slow low-energy dynamics is described quasiclassically in terms of the pair of canonically conjugated variables: the angle θ , which specifies the direction of the linear momentum of the propagating fermion, and impact parameter (or angular momentum L_z). In this description the fermion zero modes are represented by one or several branches $E_a(L_z, \theta) = -\omega_a(\theta)(L_z - L_{za}(\theta))$ of the fermionic spectrum. They cross zero energy as a function of the continuous angular momentum L_z . The number of fermion zero modes does not depend on θ and is determined by the vortex winding number N [5]: it is $2N$ if the spin degrees of freedom are taken into account. The system of these chiral fermions form the set of the one dimensional Fermi-liquids.

The final step in the calculation of the quantum spectrum of the fermions in the asymmetric core is the quantization of the remaining slow motion. It is obtained by the Bohr-Sommerfeld rule for the canonically conjugated variables [10, 11]. It appears that even for the asymmetric vortex the main structure of the quantum energy levels remains robust. One obtains one or several branches of the discrete energy levels. On each branch the levels are equidistant as in the simplest case of the axisymmetric vortex in s -wave superconductor.

Here we show that the Berry phase is important for quantization. Due to the Berry phase the bound states of fermions in the singly quantized vortex in the chiral layered superconductor with the symmetry of superfluid $^3\text{He-A}$ (possible candidate is superconducting Sr_2RuO_4 [12]) is $E = n\omega_0$, while the s -wave superconductor one has $E = (n + 1/2)\omega_0$. Thus the spectrum of the p -wave vortex contains the state with exactly zero energy, which confirms the microscopic calculations in [1]. We also consider the effect of impurities on the spectrum of bound state. We reproduce the Larkin-Ovchinnikov result for single impurity in s -wave vortex [2, 3] and extend it to the p -wave case.

2 Boundary conditions for quantization of bound states in the core of s -wave vortex.

Let us start with the N -quantum nonaxisymmetric vortex in conventional superconductors, where the asymmetry is caused, say, by crystal potential or by external perturbation. In the semiclassical approximation the quasiparticle states are characterized by their trajectories. Trajectories are straight lines along the direction of the linear momentum in transverse plane \mathbf{q}_\perp , with the total momentum being on the Fermi surface, $q_z^2 + \mathbf{q}_\perp^2 = p_F^2$. Let us consider somewhat more general case of distorted vortex in s -wave superconductor. The order parameter is chosen as $\Delta(r, \phi) = |\Delta(r, \phi)|e^{i\Phi(\phi)}$. Here ϕ is azimuthal angle; $\Phi(\phi)$ is the phase of the order parameter, which winding around the vortex is

$$N = \frac{1}{2\pi} \int_0^{2\pi} d\phi \frac{d\Phi}{d\phi}, \quad \Phi(\phi) = N\phi + \tilde{\Phi}(\phi), \quad (2)$$

where $\tilde{\Phi}(\phi)$ is a periodic function of ϕ . The Bogoliubov Hamiltonian for quasiparticles in the vicinity of such a vortex is

$$\mathcal{H} = \mathbf{p} \cdot \mathbf{v}\tau_3 + \tau_1|\Delta(r, \phi)|\cos\Phi(\phi) + \tau_2|\Delta(r, \phi)|\sin\Phi(\phi), \quad \mathbf{v} = \frac{\mathbf{q}_\perp}{m}. \quad (3)$$

Let us first neglect the conventional scattering on vortices, leaving only the Andreev reflection. In this case, in the quasiclassical limit there is no transition between different trajectories: quasiparticle is moving along the same trajectory, i.e. with the same \mathbf{q}_\perp , and experience the Andreev scattering which does not change the momentum, but changes the sign of the group velocity. Each trajectory is characterized by the angle θ in the transverse plane and by the impact parameter b :

$$b = r \sin \phi, \quad \hat{\mathbf{q}}_\perp = \hat{\mathbf{x}} \cos \theta + \hat{\mathbf{y}} \sin \theta. \quad (4)$$

Let us make two transformations of the Eq.(3) for given \mathbf{q}_\perp . The first one is the coordinate transformation which orients the axis x along the trajectory \mathbf{q}_\perp :

$$\mathcal{H} = -iv\tau_3\partial_s + \tau_1|\Delta(r, \phi - \theta)|\cos\Phi(\phi - \theta) + \tau_2|\Delta(r, \phi - \theta)|\sin\Phi(\phi - \theta). \quad (5)$$

In a new coordinate system $p_x = -i\partial_s$, where $s = r \cos \phi$ is the coordinate along the trajectory.

The second transformation is intended to make the phase of the order parameter the periodic function of θ :

$$\Psi \rightarrow e^{iN\tau_3\theta/2}\Psi, \quad (6)$$

$$\mathcal{H} \rightarrow e^{iN\tau_3\theta/2}\mathcal{H}e^{-iN\tau_3\theta/2} = \quad (7)$$

$$-iv\tau_3\partial_s + \tau_1|\Delta(r, \tilde{\phi})|\cos(\tilde{\Phi}(\tilde{\phi}) + N\phi) + \tau_2|\Delta(r, \tilde{\phi})|\sin(\tilde{\Phi}(\tilde{\phi}) + N\phi), \quad (8)$$

$$\tilde{\phi} = \phi - \theta. \quad (9)$$

Note that for the axisymmetric vortex there is no more dependence on θ :

$$\mathcal{H} = -iv\tau_3\partial_s + \tau_1|\Delta(r)|\cos(N\phi) + \tau_2|\Delta(r)|\sin(N\phi). \quad (10)$$

The quantity $N\theta/2$ plays the part of the Berry phase: when one changes the direction of the trajectory, the only change in the wave function is multiplication by $e^{iN\tau_3\theta/2}$. In more general case of nonsymmetric vortex this Berry phase is the adiabatic change in the phase of the wave function, when one continuously rotates the trajectory of the quasiparticle by the angle θ in the transverse plane.

Quantization of the motion along s gives rise to generally N fermion zero modes of the type:

$$E(L_z, \theta) = -\omega_0(\theta)(L_z - L_z^{(0)}(\theta)) \quad (11)$$

The next step is quantization of the azimuthal motion. Here it is important that the wave function in terms of θ has the boundary condition

$$\Psi(\theta + 2\pi) = (-1)^N \Psi(\theta) \quad (12)$$

which follows from Eq.(6).

The shift $L_z^{(0)}(\theta)$ from zero is antisymmetric because of the "CPT"-theorem:

$$L_z^{(0)}(\theta) = -L_z^{(0)}(\theta + \pi) \quad (13)$$

Below Eqs(11,13) will be explicitly shown for a weakly asymmetric vortex.

The quantum Hamiltonian for the canonically conjugated variables $L_z = -i\partial_\theta$ and θ is

$$\mathcal{H} = -\frac{1}{2} \left\{ \omega_0(\theta), \left(-i\frac{\partial}{\partial\theta} - L_z^{(0)}(\theta) \right) \right\}, \quad (14)$$

where $\{ , \}$ is anticommutator. The normalized wave function of the eigen state with the energy E is

$$\Psi(\theta) = \left\langle \frac{1}{\omega_0(\theta)} \right\rangle^{-1/2} \frac{1}{\sqrt{2\pi\omega_0(\theta)}} \exp \left(i \int^\theta d\theta' \left(\frac{E}{\omega_0(\theta')} + L_z^{(0)}(\theta') \right) \right) . \quad (15)$$

The energy eigenvalues $\mathcal{H}\Psi_n(\theta) = E_n\Psi_n(\theta)$ are found from the requirement that according to Eq.(12) the wave function $\Psi_n(\theta)$ changes sign after encircling the origin in the momentum space θ for odd N and does not change if N is even, i.e. the phase of the wave function changes by $\pi(2n + N)$. This gives

$$E_n \int_0^{2\pi} \frac{d\theta}{\omega_0(\theta)} + \int_0^{2\pi} d\theta L_z^{(0)}(\theta) = -2\pi \left(n + \frac{N}{2} \right) . \quad (16)$$

The second term in the lhs of Eq.(16) is zero according to Eq.(13), and one obtains the quantization which is essentially the same as for the axisymmetric vortex:

$$E_n = -\frac{n + \frac{1}{2}}{\left\langle \omega_0^{-1}(\theta) \right\rangle} , \quad \text{odd } N \quad (17)$$

$$E_n = -\frac{n}{\left\langle \omega_0^{-1}(\theta) \right\rangle} , \quad \text{even } N. \quad (18)$$

For even N there is a state with zero energy.

3 Fermion zero modes in a weakly asymmetric vortex with odd N in s -wave superconductors.

Let us consider a weakly asymmetric vortex in which the anisotropy is small, say, $|\tilde{\Phi}(\tilde{\phi})| \ll 1$. In the extreme limit, when $b = 0$ and $\tilde{\Phi}(\tilde{\phi}) = 0$, the trajectory with $\phi = 0$ for $s > 0$ and $\phi = \pi$ for $s < 0$ gives exactly zero energy if N is odd. This is because the order parameter along this trajectory is real and changes sign when the origin is crossed. The index theorem requires that there is the normalizable eigen state with zero energy.

Let us choose an odd N , then for small b the perturbation theory can be constructed. Since ϕ is close to 0 or π one has

$$\tilde{\phi} = \phi - \theta \approx \frac{\pi}{2} (1 - \text{sign } s) - \theta . \quad (19)$$

and

$$\mathcal{H} = \mathcal{H}^{(0)} + \mathcal{H}^{(1)} , \quad (20)$$

$$\mathcal{H}^{(0)} = -iv\tau_3 \frac{d}{ds} + \tau_1 |\Delta(s, \theta)| \text{sign } s , \quad (21)$$

$$\Delta(s, \theta) = \Delta \left(|s|, \frac{\pi}{2} (1 - \text{sign } s) - \theta \right) \quad (22)$$

$$\mathcal{H}^{(1)} = \tau_2 |\Delta(s, \theta)| \sin \left(\tilde{\Phi}(\tilde{\phi}) + N\phi \right) \approx \quad (23)$$

$$\approx \tau_2 |\Delta(s, \theta)| \left(N \frac{b}{|s|} + (\text{sign } s) \tilde{\Phi} \left(\frac{\pi}{2} (1 - \text{sign } s) - \theta \right) \right) . \quad (24)$$

The Hamiltonian $\mathcal{H}^{(0)}$ has fermion zero mode, $\Psi^{(0)}(s)$, so that the energy is the average of $\mathcal{H}^{(1)}$ over this mode:

$$E(b, \theta) = \left\langle \mathcal{H}^{(1)} \right\rangle_s = -Nb \left\langle \frac{|\Delta(|s|, -\theta)| + |\Delta(|s|, \pi - \theta)|}{2|s|} \right\rangle_s + \quad (25)$$

$$+ \left\langle |\Delta(|s|, -\theta)| \tilde{\Phi}(-\theta) - |\Delta(|s|, \pi - \theta)| \tilde{\Phi}(\pi - \theta) \right\rangle_s . \quad (26)$$

This gives the Eq.(11) for the quasiclassical energy $E(L_z, \theta)$ and the Eq.(13) for the shift $L_z^{(0)}(\theta)$. The Eq.(26) is $E(0, \theta) = \omega_0(\theta) L_z^{(0)}(\theta)$: it is the energy of quasiparticle moving along trajectory with $b = 0$. The nonzero value of this energy comes from the imaginary part of the order parameter $\text{Im } \Delta(|s|, -\theta)$ on the trajectory.

4 Boundary conditions for quantization of bound states in the vortex core in chiral superconductor.

Let us consider simplest case of the axisymmetric vortex in p -wave superconductor with the $^3\text{He-A}$ order parameter. The order parameter in this vortex

is $c(r)(p_x + ip_y)e^{iN\phi}$, where $c = \frac{\Delta_0}{p_F}$ and Δ_0 is the gap amplitude. The Bogoliubov Hamiltonian for quasiparticles in the vortex core along the trajectory with given θ is

$$\mathcal{H} = \mathbf{p} \cdot \mathbf{v}\tau_3 + \tau_1 q_\perp c(r) \cos(\theta + N\phi) + \tau_2 q_\perp c(r) \sin(\theta + N\phi) , \quad c(r) = \frac{\Delta_0(r)}{p_F} . \quad (27)$$

The coordinate transformation which orients the axis x along the trajectory \mathbf{q}_\perp :

$$\frac{\mathcal{H}}{q_\perp} = -i\frac{1}{m}\tau_3\partial_s + \tau_1 c(r) \cos(N\phi - (N-1)\theta) + \tau_2 c(r) \sin(N\phi - (N-1)\theta) . \quad (28)$$

The second transformation is intended to delete the dependence on θ :

$$\Psi \rightarrow e^{i(N-1)\tau_3\theta/2}\Psi , \quad (29)$$

$$e^{i(N-1)\tau_3\theta/2}\mathcal{H}e^{-i(N-1)\tau_3\theta/2} = q_\perp \left(-\frac{i}{m}\tau_3\partial_s + \tau_1 c(r) \cos N\phi + \tau_2 c(r) \sin N\phi \right) . \quad (30)$$

The new Hamiltonian is the same as for the N -quantum symmetric vortex in s -wave superconductor, but the Berry phase is different, now it is $(N-1)\tau_3\theta/2$. This leads to different boundary condition for the wave function, which according to Eq.(29) is

$$\Psi(\theta + 2\pi) = (-1)^{N+1}\Psi(\theta) \quad (31)$$

This leads to the following quantization of levels in the core of the p -wave vortex with odd N :

$$E_n = -n\omega_0 , \quad \text{odd } N . \quad (32)$$

Thus for $N = \pm 1$ vortex there is a zero energy level.

In general case the Cooper pair has an angular momentum projection m . Before we considered two examples: the s -wave superconductor, which belongs to the class $m = 0$; and the A-phase-like superconductors, which belongs to class $m = \pm 1$. In the case of general m , the Berry phase and boundary condition for the wave function of fermions in N -quantum vortex are

$$\Theta_{\text{Berry}} = \frac{N-m}{2}\tau_3\theta , \quad \Psi(\theta + 2\pi) = (-1)^{N-m}\Psi(\theta) \quad (33)$$

This leads to two classes of the fermionic spectrum in the symmetric core: $E_n = n\omega_0$ if $N-m$ is even and $E_n = (n+1/2)\omega_0$ if $N-m$ is odd.

5 Effect of impurity scattering.

Elastic scattering on impurity causes the transition between different trajectories. In the limit of low energy of the quasiparticle the impact parameter of the scattered particle tends to zero and becomes smaller than the distance from impurity to the center of the vortex. If the size of impurity is small then the scattering of the low-energy quasiparticle occurs only between the trajectories along the line between the vortex and impurities, i.e. between $\theta = \theta_{\text{imp}}$ and $\theta = \pi - \theta_{\text{imp}}$ [13]. The "Josephson coupling" between these two states is

$$\mathcal{H}_{\text{imp}} = 2\lambda e^{i\gamma} \Psi(\pi - \theta_{\text{imp}}) \Psi^*(\theta_{\text{imp}}) + 2\lambda e^{-i\gamma} \Psi^*(\pi - \theta_{\text{imp}}) \Psi(\theta_{\text{imp}}) . \quad (34)$$

The Schrödinger equation is now

$$-i\omega_0 \frac{\partial \Psi}{\partial \theta} + 2\lambda e^{i\gamma} \delta(\theta - \theta_{\text{imp}}) \Psi(\pi - \theta_{\text{imp}}) + 2\lambda e^{-i\gamma} \delta(\theta - \pi + \theta_{\text{imp}}) \Psi(\theta_{\text{imp}}) = E \Psi(\theta) , \quad (35)$$

Let us choose the position of impurity at $\pi/2$. Then the relevant trajectories, where the scattering occurs, are at $\pm\pi/2$ and one has

$$-i\omega_0 \frac{\partial \Psi}{\partial \theta} + 2\lambda e^{i\gamma} \delta\left(\theta - \frac{\pi}{2}\right) \Psi\left(-\frac{\pi}{2}\right) + 2\lambda e^{-i\gamma} \delta\left(\theta + \frac{\pi}{2}\right) \Psi\left(\frac{\pi}{2}\right) = E \Psi(\theta) , \quad (36)$$

with boundary condition

$$\Psi(-\pi) = \pm \Psi(\pi) \quad (37)$$

Here the sign $+$ is for the p -wave $N = 1$ vortex and $-$ is for s -wave $N = 1$ vortex. The solution is

$$\Psi = A_1 e^{i\frac{E}{\omega_0}\theta} , \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} , \quad (38)$$

$$\Psi = A_2 e^{i\frac{E}{\omega_0}\theta} , \quad -\pi < \theta < -\frac{\pi}{2} , \quad (39)$$

$$\Psi = A_3 e^{i\frac{E}{\omega_0}\theta} , \quad \frac{\pi}{2} < \theta < \pi . \quad (40)$$

The conditions across two δ -function potentials and the boundary condition Eq.(37) give 3 equations for the parameters A :

$$-i\omega_0 e^{-i\frac{\pi E}{2\omega_0}} (A_1 - A_2) = \lambda e^{i\gamma} e^{i\frac{\pi E}{2\omega_0}} (A_1 + A_3) , \quad (41)$$

$$-i\omega_0 e^{i\frac{\pi E}{2\omega_0}}(A_3 - A_1) = \lambda e^{-i\gamma} e^{-i\frac{\pi E}{2\omega_0}}(A_1 + A_2) , \quad (42)$$

$$e^{i\frac{\pi E}{\omega_0}} A_3 = \pm e^{-i\frac{\pi E}{\omega_0}} A_2 . \quad (43)$$

Solution of these equations give the energy eigenvalues:

$$\cos \frac{\pi E}{\omega_0} = \frac{2\omega_0 \lambda}{\omega_0^2 + \lambda^2} \sin \gamma , \quad s - \text{wave} , \quad (44)$$

$$\sin \frac{\pi E}{\omega_0} = \frac{2\omega_0 \lambda}{\omega_0^2 + \lambda^2} \cos \gamma , \quad p - \text{wave} . \quad (45)$$

In the s -wave case the Eq.(44) is similar to Eq.(2.10) of Ref.[3]: the spectrum in the presence of impurity has the double period $\Delta E = 2\omega_0$ and consists of two equidistant sets of levels. These two sets transform to each other under symmetry transformation $E \rightarrow -E$, which is the "CPT"-symmetry of the system.

For the p -wave case the Eq.(45) also gives two sets of levels with the alternating shift. But the two sets are not mutually symmetric with respect to $E = 0$. This contradicts to the "CPT"-symmetry of the system. The only way to reconcile the Eq.(45) with the symmetry is to fix the phase $\gamma = \pi/2$ of the "Josephson coupling". Then the energy levels are $E_n = n\omega_0$, i.e. the same as without impurities. Thus the same "CPT"-symmetry, which is responsible for the eigenstate with $E = 0$, provides the rigidity of the spectrum.

6 Conclusion

The Berry phase in Eq.(33) is instrumental for the Bohr-Sommerfeld quantization of the energy levels in the vortex core. It chooses between the two possible quantizations consistent with the CPT -symmetry of states in superconductors: $E_n = n\omega_0$ and $E_n = (n + 1/2)\omega_0$.

We found that the two spectra remain intact if the small perturbation is added, which violates the axial symmetry of the order parameter. We also considered the effect of single impurity on the spectrum of bound states. We found that if in a pure superconductor the spectrum is $E_n = n\omega_0$ (an example is $N = 1$ vortex in chiral superconductor with $m = 1$) then the impurity does not change this spectrum. On the other hand if the initial spectrum

is $E_n = (n + 1/2)\omega_0$, the impurity splits it into two series according to the Larkin-Ovchinnikov prescription [2, 3]. This rigidity of the spectrum must be taken into account when the effect of randomness due to many impurities is considered with introduction of new level statistics for the fermionic spectrum in the core [14].

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